Similarity Metrics for Bounding Volumes

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Testing the similarity of bounding volumes is a common operation in geometry processing. Often, the intersection or union of two bounding volumes is calculated as an intermediate step. These intermediate volumes are then measured to produce a comparison metric. Common metrics include distance, surface area, volume and overlap.

Here, we introduce new variations of these metrics, using a new intermediate calculation inspired by interval arithmetic. We argue that the new technique offers better precision than calculating intersections or unions of bounding volumes because more detail is available over the entire range of inputs.

Regarding notation, we write a vector as $\underline{V}$; its component in dimension i is given with v_i. An axis-aligned bounding box, R_0, has a minimum coordinate of r_0^{i-} and a maximum coordinate of r_0^{i+} in dimension i.

For our comparison metrics, we introduce two new vectors, $\underline{M}$ and $\underline{N}$, whose components are calculated as a function of two axis-aligned bounding boxes, R_0 and R_1:

$$m_i = \max(\, r_0^{i+} - r_1^{i-},\ r_1^{i+} - r_0^{i-} \,)$$
$$n_i = \min(\, r_0^{i+} - r_1^{i-},\ r_1^{i+} - r_0^{i-} \,)$$

These calculations are easily adapted for bounding spheres and other volume types.

Intuitively, $\underline{M}$ varies with the edge length of the union between the two volumes. When one volume fully contains the other; however, the edge length of the union would clamp to the larger of the volumes. With $\underline{M}$, containment does not cause clamping; $\underline{M}$ will continue to vary with the relative sizes and positions of the volumes. This is why we say that $\underline{M}$ has greater precision than the edge length of the union of the bounding volumes.

Similarly, $\underline{N}$ varies with the edge length of the intersection of the two volumes. As with $\underline{M}$, containment does not cause $\underline{N}$ to clamp. Also, the components of $\underline{N}$ do not clamp to zero when the inputs do not overlap. Therefore, we say that $\underline{N}$ has greater precision than the edge length of the intersection of the bounding volumes.

We are able to make several useful observations:

Edge Length Property: $(m_i + n_i) = (r_0^{i+} - r_0^{i-}) + (r_1^{i+} - r_1^{i-})$

Overlap Property: if $n_i < 0$ then R_0 and R_1 don't overlap in dimension i.

Centers Property: if the intervals of R_0 and R_1 in dimension i have equal midpoints, then $m_i = n_i$, otherwise $m_i > n_i$.

Volume Property: if both R_0 and R_1 have zero edge length and exist at the same coordinate in dimension i, then $m_i = 0$; otherwise $m_i > 0$.

From these properties, it is clear that the corresponding components of $\underline{M}$ and $\underline{N}$ form intervals which describe the relative relationship between the input intervals. To produce a scalar value suitable for comparison, these intervals must be collapsed. We present several options:

Furthest (Minkowski) Distance Metric:

$$f_1(R_0, R_1, k) = \left(\sum_i m_i^{\,k} \right)^{1/k}$$

Combined Surface Area Metric: $f_2(R_0, R_1) = \sum_s \left(\prod_{t \neq s} m_t \right)$

Combined Volume Metric: $f_3(R_0, R_1) = \prod m_i$

Shared Volume Metric: $f_4(R_0, R_1) = \sum n_i m_i$

Relative Proximity of Centers Metric:

$f_5(R_0, R_1) = \sum \dfrac{n_i}{m_i + \varepsilon}$, where ε is a very small positive value.

Absolute Proximity of Centers Metric:

$f_6(R_0, R_1) = \sum n_i - m_i$

References

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